String Theory: How do Musicians Know what Sounds Nice?

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Why talk about music?

"Music makes us feel things, lyrics make us think things, and songs makes us feel thoughts"

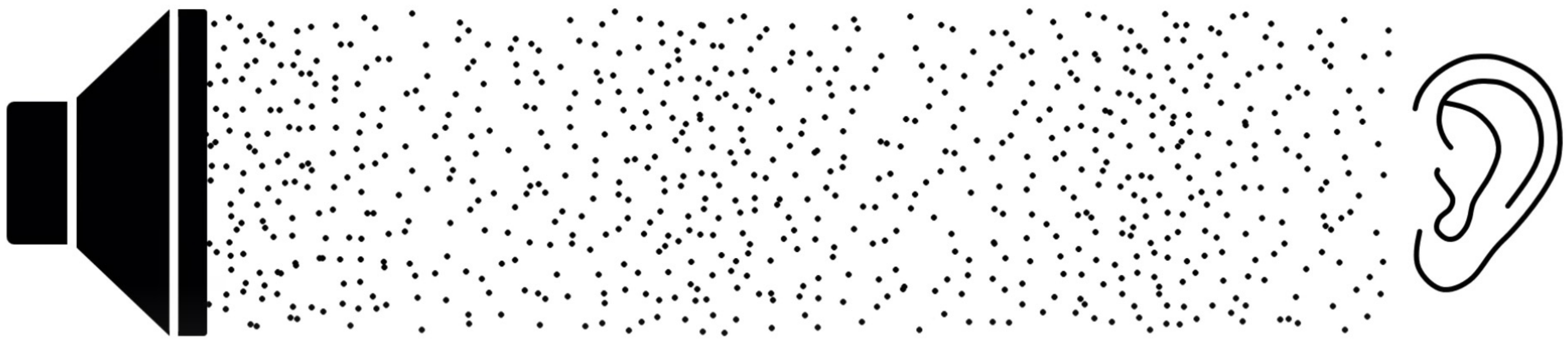
- Someone on YouTube

Prerequisites & Goals ❀

- Modular Arithmetic
  - i.e. count to seven then repeat
- Knowledge of what a function is
- Solve partial differential equations
- Explain how sound is produced and how we biologically interpret it
- Know 12 notes in western music
- Recite pattern for major and minor scale
  - What notes $\in$ scale & chords $\in$ key
- Explain what notes make up a major/minor chord

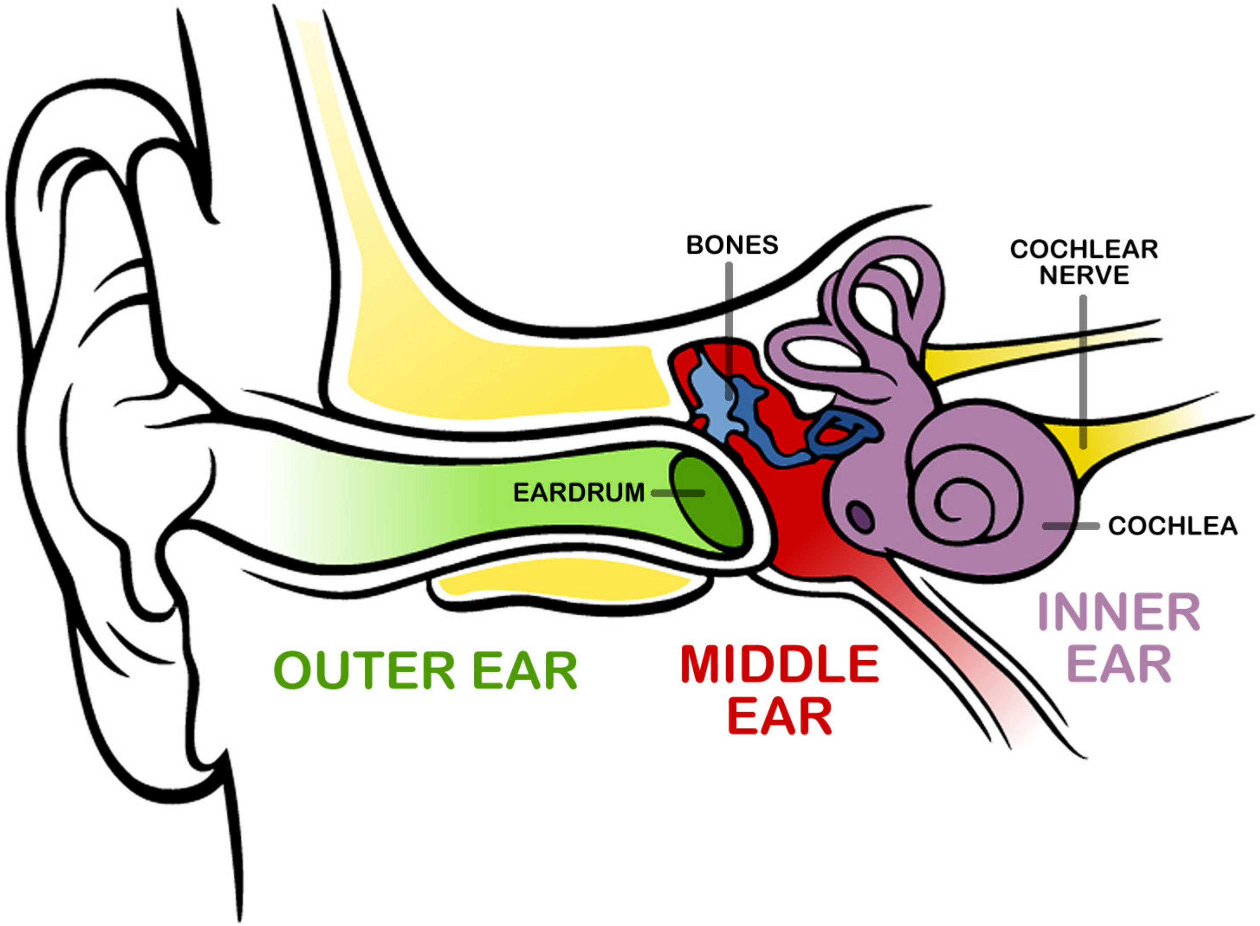
What is Sound?

- Vibration that moves through some medium
- Frequency (Hz) is no. of vibrations per second
- We interpret a higher frequency as a higher pitch

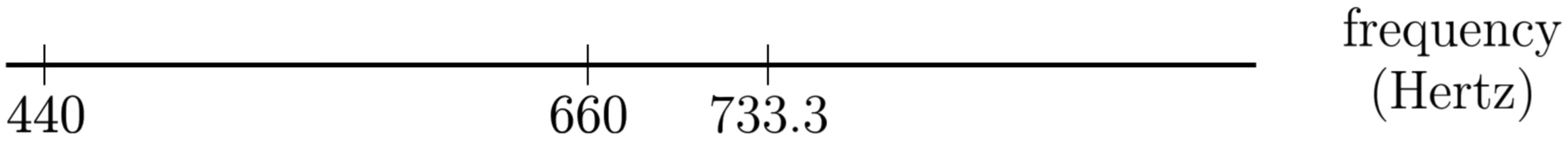


Frequency (in Hertz) = no. of vibrations per second

Some Biology ❀



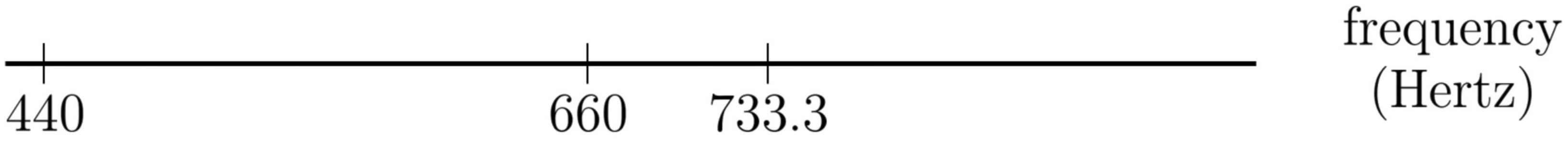
Melody I



Melody II

Melody I

$$\frac{660}{440} = \frac{3}{2} \qquad \frac{733.3}{440} = \frac{5}{3}$$



Melody II

$$\frac{825}{550} = \frac{3}{2} \qquad \frac{916.6}{550} = \frac{5}{3}$$

Wave Equation (for the math nerds)

- $\frac{\partial^2 u}{\partial t^2}(x,t) = c^2 \frac{\partial^2 u}{\partial x^2}(x,t)$
- What is u(x, t) if we assume $u(x,t) = y(x) \cdot z(t)$?

Wave Equation (for the math nerds)

- $\frac{\partial^2 u}{\partial t^2}(x,t) = c^2 \frac{\partial^2 u}{\partial x^2}(x,t)$
- Answer: **sin(nx)** and some other terms depending on boundary conditions
- If curious: https://personal.math.ubc.ca/~feldman/m267/separation.pdf

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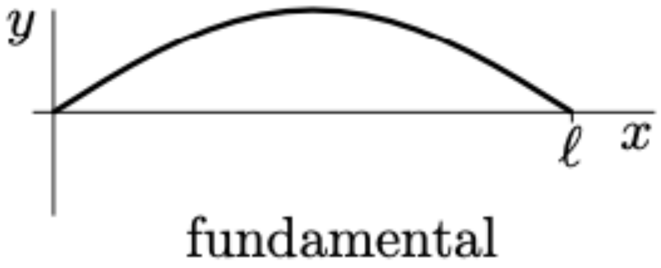
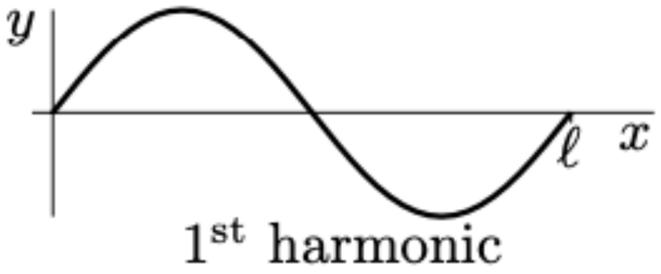
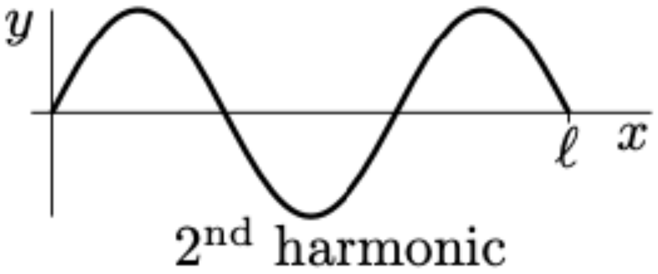
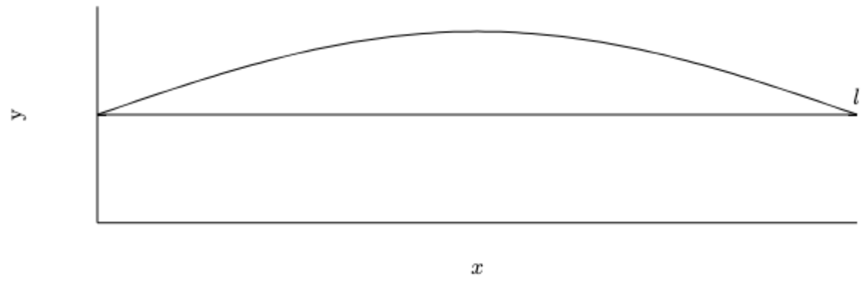






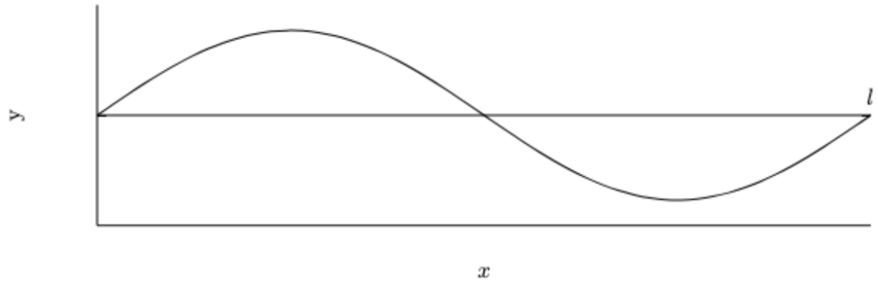
n = 1



Fundamental

f_1

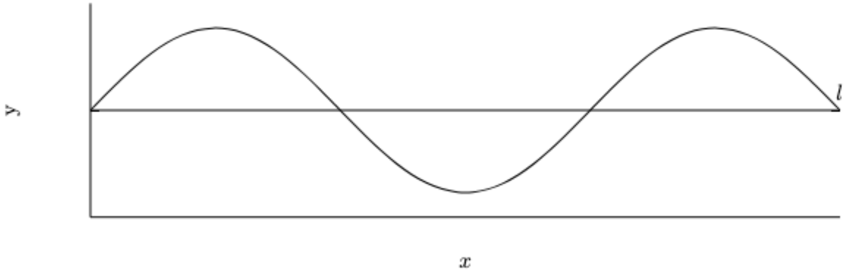
n = 2



1st Harmonic

$2 \bullet f_1$

n = 3



2nd Harmonic

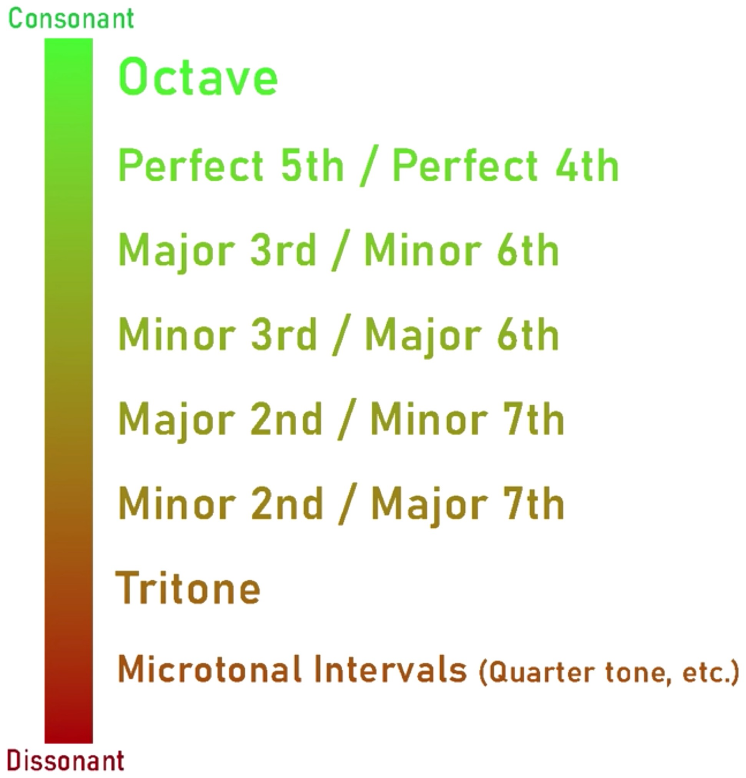
$3 \bullet f_1$

"Postulates" of Western Music Theory

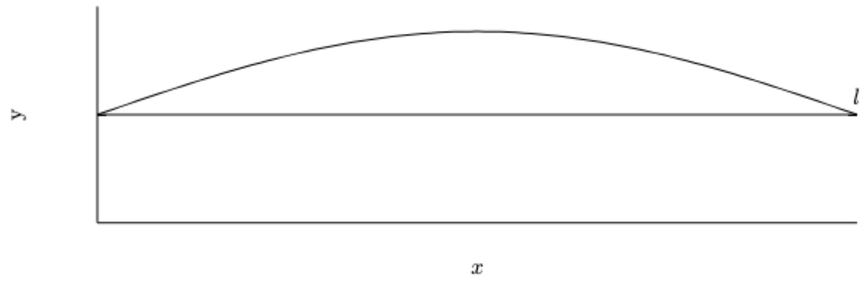
- 12 Tone Equal Temperament
  - A, A#/Bb, B, C, C#/Db, D, D#/Eb, E, F, G, G#/Ab ❀
  - Smallest possible increment is a "half-step" if you're from North America and a "semi-tone" if you're from Europe
- Major Scale :
  - W W H W W W H
  - Ex. C Major: C D E F G A B C
- Minor Scale :
  - W H W W H W W
  - Ex. C Minor: C D Eb F G Ab Bb C

Why these 12 Notes?

- Subjective but general agreement



n = 1

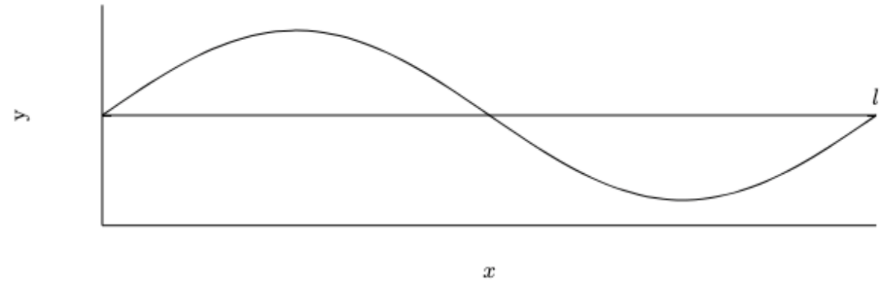


Fundamental

f_1

Octave

n = 2

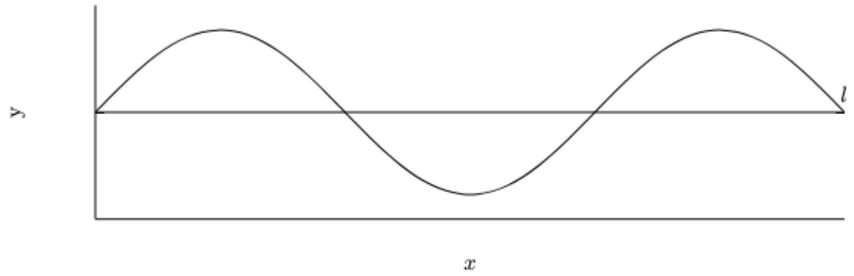


1st Harmonic

$2 \bullet f_1$

Perfect 5th

n = 3



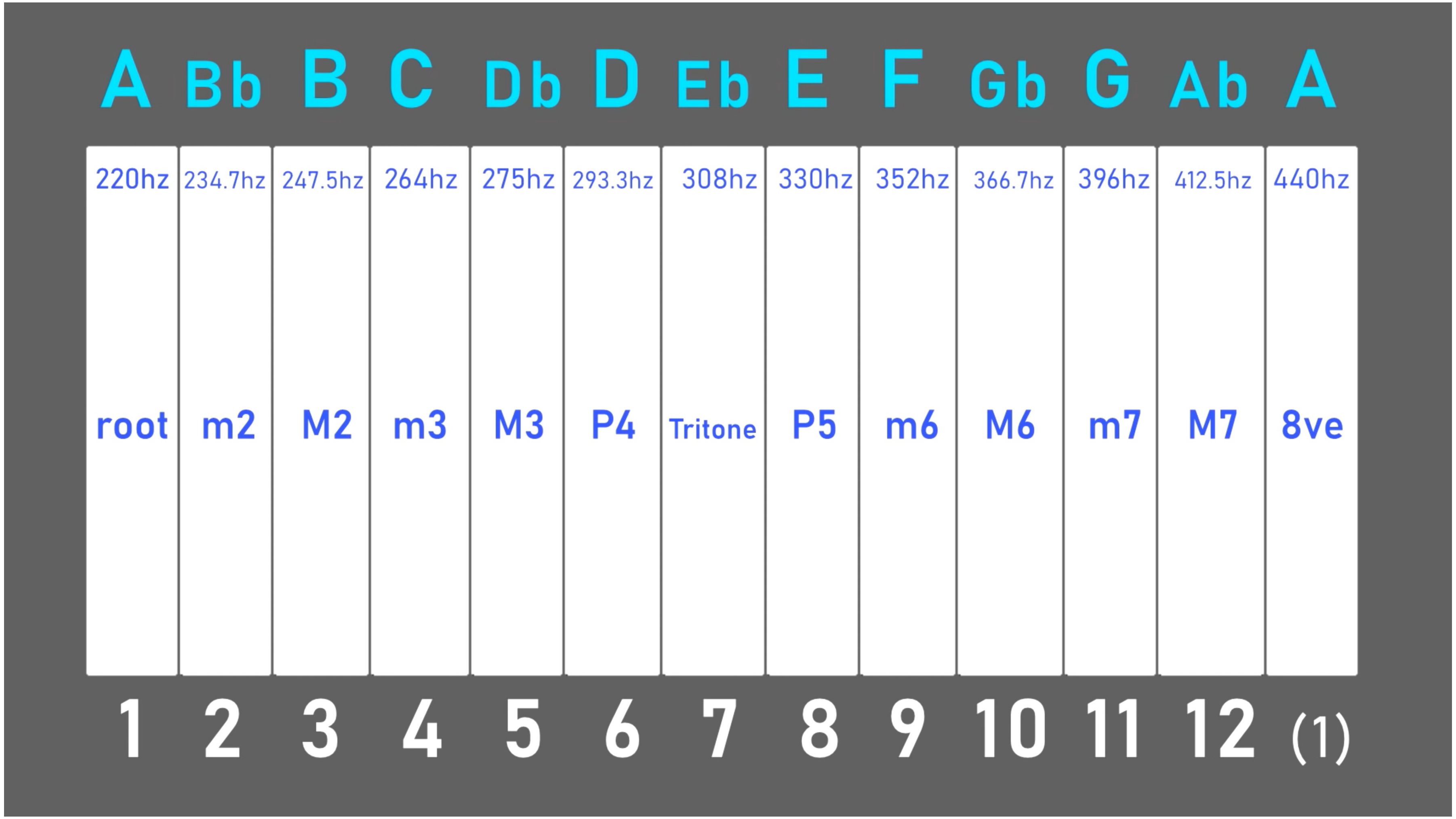
2nd Harmonic

$3 \bullet f_1$

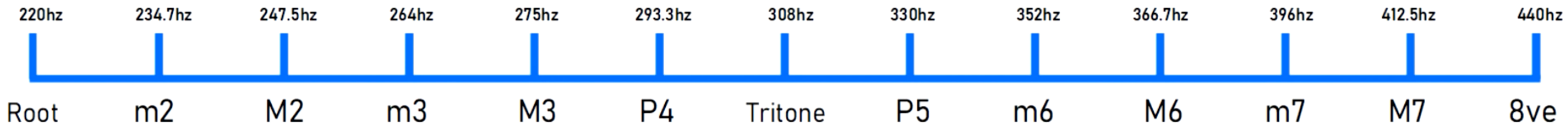
Summary of Intervals

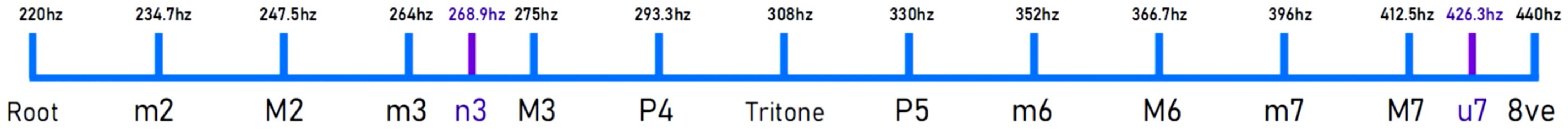
Interval	Relationship between the notes
Octave	2:1
Perfect Fifth	3:2
Perfect Fourth	4:3
Major Third	5:4
Minor Third	6:5
Major Sixth	8:5
Minor Sixth	5:3
Major Seventh	15:8
Minor Seventh	9:5
Tritone	7:5
Major Second	9:8
Minor Second	16:15

Arbitrary “Coordinate System”

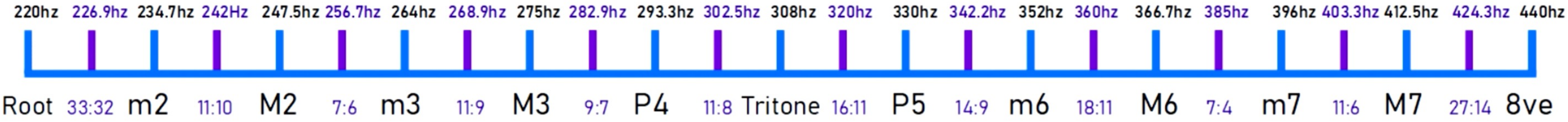


Why not add intermediate frequencies?

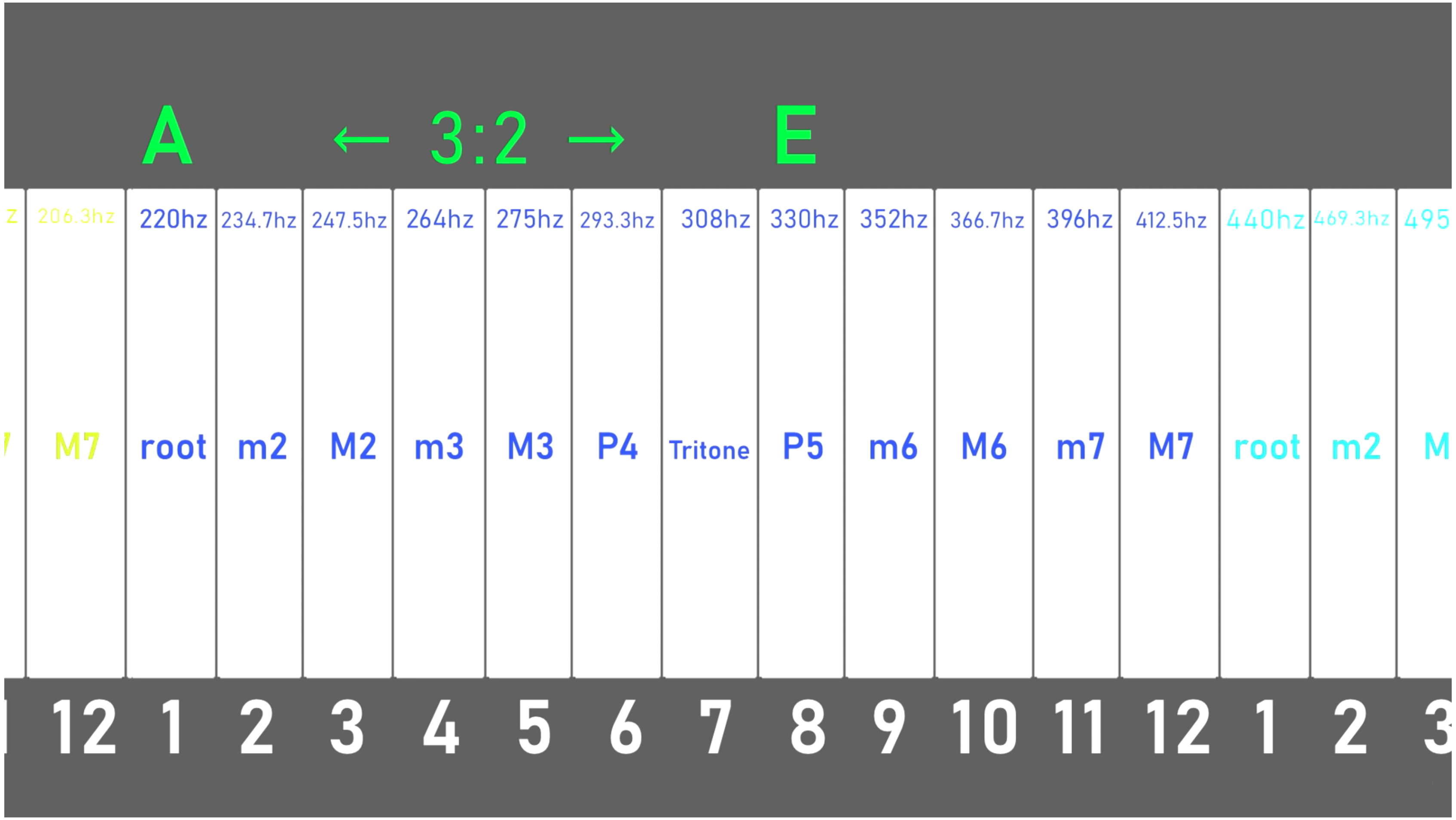




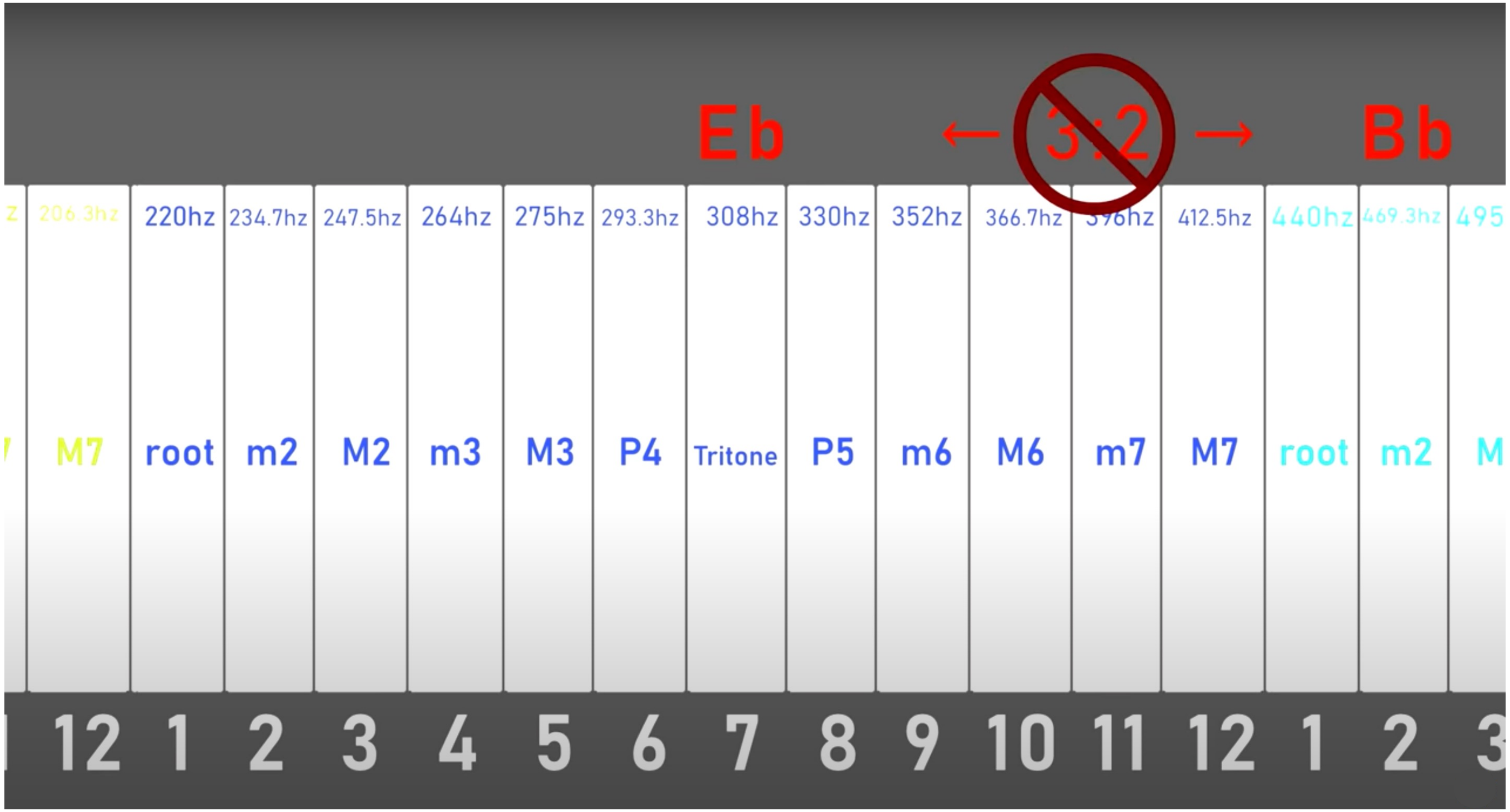
What if Evenly Spaced?



Where Just Intonation Fails



Where Just Intonation Fails



Framed Another Way

- If we try to tune using these perfect intervals:
- 6 Major 2nd's in an octave
  - $(9/8)^6 = 2.027\ldots \neq 2$
- 3 Major 3rd's in an octave
  - $(5/4)^3 = 1.95\ldots \neq 2$
- 12 Half Steps
  - $(16/15)^{12} = 2.17\ldots \neq 2$
- Hold true for any of these intervals

Interval	Relationship between the notes
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Major Seventh	15:8
Minor Seventh	9:5
Tritone	7:5
Major Second	9:8
Minor Second	16:15

Solution: Slightly Detune

- Focus on the octave as the most important interval, and tune every other note using $2^{n/12}$ for the nth note
- $2^{1/12}$ = 1.059 ~ 16/15 = 1.066… (half step)
- $2^{2/12}$ = 1.112 ~ 9/8 = 1.125… (whole step)
- …
- $2^{12/12}$ = 2 = 2/1 (octave)
- This also gives a nice explanation for why it is natural to have 12 notes in a chromatic scale, and not some other number: powers of the twelfth root of two have a tendency to be surprisingly close to simple rational numbers!

12 TET

Interval	Just Intonation compared with 12TET (cent)
Octave	0
Perfect Fifth	+1.96
Perfect Fourth	-1.96
Major Third	-13.69
Minor Third	+15.64
Major Sixth	-15.64
Minor Sixth	+13.69
Major Seventh	-11.73
Minor Seventh	+17.6
Tritone	-17.49
Major Second	+3.91
Minor Second	+11.73

Actual Music Theory

- Major Scale
  - Root, M2, M3, P4, P5, M6, M7, Root
- Major Key
  - I, ii, iii, IV, V, vi, dim(vii), I
- Minor Scale
  - Root, M2, m3, P4, P5, M6, m7, Root
- Minor Key
  - i, dim(ii), III, iv, v, VI, VII

Chords

- A chord is made up of three notes: root, 3rd, and 5th
  - If it's a major chord: root, M 3rd, 5th
  - If it's a minor chord: foot, m 3rd, 5th
- A very subjective notion, but there are certain feels associated with the intervals between notes
- Sometimes musicians refer to these feelings as colors where something more colorful has a greater sense of tension and release)

Chord Progressions

- A chord progression is a sequence of chords in an associated key
- Some Common Ones are as Follows:
- Axis of Awesome (pop songs): I, V, vi, IV
- Jazz: ii, V, I
- 12 Bar Blues

Acknowledgements

- Thank you to everyone who attended this talk! Especially those who found it even the slightest bit interesting and/or pleasant.

"Beauty will save the world."

\- Fyodor Dostoevsky